VARIABILITY PROPAGATION IN PERISHABLE PRODUCT SUPPLY CHAINS

Abstract. Demand variability and its propagation in the supply chain have played a key role in recent shortages, inflation and turmoil. Managing demand variability is essential to minimizing costs and delivering reliable supply. In perishable-product supply chains it is also key to reducing food waste and carbon emissions. This study provides the first analysis of variability propagation in perishable-product supply chains. We build and calibrate a two-echelon perishable-inventory model, showing that the nature of variability propagation in perishable-product supply chains is strikingly different from that in well-studied durable-product supply chains. In particular, we find that (i) product perishability is a novel, hitherto unknown driver of the much-examined bullwhip effect (upstream variability amplification), (ii) surprisingly, perishability can also lead to upstream variability attenuation, an *anti*-bullwhip effect. Our data-driven model calibration reveals a great variation in the degree of variability amplification across different products resulting from more/less favorable combinations of the product and market characteristics. Products with more extreme (high or low) purchasing price, replenishment cost, mean of product expiration time, and standard deviation of buyer demand exhibit higher variability amplification. High mean of buyer demand and standard deviation of product expiration time also yield higher amplification. Finally, we show that the buyer's order quantity modulates the extent of the upstream variability amplification, and as a result, the supply chain partners could attempt to identify contracts that coordinate on buyer's order quantities to limit variability amplification. This could lead to overall less food waste (3-6%) and higher profits (2-10%) for the supply chain.

1. Introduction

Responding to variability is one of the main objectives of modern operations management. Variability in demand for a product propagates through its supply chain – variability in demand faced by a buyer leads to variability in their orders to a supplier, which is variability in demand faced by a supplier. The same applies to the supplier with respect to firms further upstream, and in this way, variability propagates up the supply chain. Variability propagation in the supply chain has been consistently identified as one of the top challenges facing supply chains (Supply Chain Quarterly, 2011; Egon Zehnder, 2020).

The Covid-19 pandemic, the Ukraine war, and other recent events highlighted the importance of understanding and managing variability propagation. Variability propagation, and the poor understanding of it, have been blamed for the ensuing persistent shortages and oversupply of assembled goods and food items, out-of-control inflation and even political turmoil (Coy, 2021; Lee, 2022; Krugman, 2022).

Across various industries, trade estimates suggest that the costs of demand variability range from 12.5% to 25% (Lee et al., 1997). The problem takes heightened importance in the grocery industry where a large part of the inventory is made of perishable products. In grocery retail, the excess inventory within the supply chain is estimated to be about 25% to 30% of total sales (Fuller et al., 1993). This excess inventory is not only a financial burden, but also ultimately results in unnecessary food waste, an even more pernicious effect. In fact, the majority of food waste (about 42%) arises from excess inventory in such retail supply chains and is a significant contributor to global warming (ReFED, 2021). As such, better management of demand variability within the perishable-product (food) supply chain can lead to reductions in excess inventory or food waste. These reductions lead to substantial financial gains, while simultaneously reducing the associated carbon emissions – a rare win-win opportunity to address global warming.

This study provides the first analysis of variability propagation in *perishable-product* supply chains. We identify (a) how variability propagates in perishable-product supply chains, (b) conditions that lead to variability amplification or attenuation, (c) the role of the market and product characteristics in variability propagation, and (d) opportunities for efficiency improvements and carbon footprint reduction through better managing demand variability propagation.

We build and calibrate a two-echelon perishable-inventory model. Ongoing customer demand is modeled as a stochastic process. To satisfy the continuing and random customer demand, the buyer buys the perishable product from the supplier and builds his inventories. In turn, the supplier replenishes the product and builds supplier inventories to satisfy buyer demand. Each buyer's replenishment point generates an arrival of the supplier demand, leading to the endogenous formation of the supplier demand.

While we observer the much-celebrated *bullwhip effect* – the phenomenon where the supplier's demand variability (or equivalently, the variability of the buyer's orders) is *higher* than the buyer's demand variability (Lee et al., 1997), our model does not include any known drivers of the bullwhip effect (demand forecast updating, order batching, price fluctuation, rationing and gaming, etc.).

As a result, for durable (non-perishable) products, we predictably observe no variability amplification/attenuation, and the bullwhip effect does not arise. That is, the variability amplification ratio (the ratio of the coefficients of variation of the supplier and the buyer demands) for durable products equals 1. The results are strikingly different for perishable products:

For perishable products, we find that even in the absence of all the known factors driving the bullwhip effect, variability may be amplified upstream (the bullwhip effect), and even more surprisingly, variability may even get attenuated upstream – an *"anti-bullwhip" effect*, the supplier's demand variability is *lower* than the buyer's demand variability. That is, the variability amplification ratio for these perishable products can be greater or lower than 1. In effect, this analysis shows that product perishability is a novel, hitherto unknown driver of variability propagation and the much examined and celebrated bullwhip effect. This new driver behaves in unexpected ways – both *amplifying* variability (as previous drivers) and also *attenuating* upstream variability, unlike previously studied drivers.

Using data from various industry reports and academic studies, we calibrate our model for a number of common perishable products to estimate the variability amplification ratios. The amplification ratios vary drastically depending on product characteristics ranging from 0.38 to as much as 4.1. These significant differences result from more/less favorable combinations of the product characteristics. Further modeling and theoretical analysis allows us to show that supply chains for products with more extreme (high or low) purchasing price, replenishment cost, mean of product expiration time, and standard deviation of buyer demand have higher variability amplification ratio. High mean buyer demand and standard deviation of product expiration time also yield higher amplification ratios.

The key to understanding the unexpected variability attenuating and amplifying effects of our new driver, product perishability, lies in understanding the dependence between the amplification ratio and the order-up-to level chosen by the buyer in managing their inventories (the buyer's order quantity). For durable products, the amplification ratio does not depend on the order quantity. In contrast, in perishable-product supply chains, the variability amplification ratio is a non-monotonic function of the buyer's order quantity; in particular, the amplification ratio is decreasing in the buyer's order quantity below a threshold and increasing above.

In perishable products, supplier demand varies due to two root causes – variability in buyer demand and the randomness in the shelf-life of a product. Buyer orders (and thus supplier demand) arise out of depleted inventories on account of random buyer demand or random expiration of products in the buyer's inventory. These two variabilities propagate upstream in different ways, moderated by the order quantity.

Consider the special case where there is no buyer demand variability: demand is deterministic while product shelf-life is still random. Now for low order quantities, in-stock inventory rarely expires and orders are driven entirely by deterministic demand and have very low variability. On the other hand, for high order quantities, orders are driven almost entirely by random product expiration, in a way inventory levels are *truncated* by the random shelf-life. Overall, this leads to higher amplification ratios that are increasing in order quantity. Next consider the alternate case where there is no random expiration time – shelf life is deterministic, while demand is random. Now for high order quantities, orders are truncated by the deterministic shelf life. For low order quantities, orders are driven by the *random* demand. Overall, this leads to amplification ratios that are decreasing in order quantity.

These special cases illustrate how shelf-life driven truncation of inventory levels leads to different variability propagation depending on the order quantity. Together, when both shelf-life and demand are random, this leads to a non-monotonic dependence of the amplification ratio on the order quantity. This non-monotonicity, combined with the observation that at very low order quantities the amplification ratio is 1 (orders directly reflect demand), leads to our central finding that for different optimal order quantities (depending on product economics and market characteristics), we can have ratios below 1 (anti-bullwhip) or above 1 (bullwhip). The relationship between different market and product characteristics and the amplification ratio now follows – these parameters lead to different optimal order quantities and different threshold quantity levels and, thus, different amplification ratios.

We close our study by examining ways in which our newly found dependence of variability propagation on order quantity could be used to improve profits. Since the extent of the upstream variability amplification is modulated by buyer's order quantity, the supply chain partners could attempt to identify contracts that coordinate on buyer's order quantities such that there is limited variability amplification. This can lead to overall higher supply chain profits and improved outcomes for all players; the supply chain could generate less food waste and higher profits. Our calibrated numerical analysis shows that there exist contracts that can manage variability propagation such that supply chain profits increase by 2-10% and food waste falls by 3-6%. With appropriate payments to redistribute gains and incentivize coordination, we can thus ensure that all supply chain players have higher profits while lowering the carbon footprint – all achieved by controlling variability propagation in supply chains.

Overall, our study provides the first analysis of variability propagation in perishable-product supply chains. We identify *product perishability as a new driver* of the bullwhip effect. This driver can lead to variability amplification (like previous drivers) or even variability attenuation (unlike previous drivers) depending on product economics and market characteristics. Finally, the existence of variability attenuation can be exploited to create coordinating mechanisms that improve the financial and environmental performance of supply chains.

These findings expand and change our current understanding of variability propagation in supply chains – one of the most well-studied supply chain phenomena. They also shed light on the empirical literature which has considered perishable and durable products alike and has struggled to identify clear effects at the industry level (Cachon et al., 2007). Our theory suggests that this analysis should be conducted at the product level while appropriately accounting for product characteristics and the perishability of the product in question. The identified coordination opportunities show a pathway for supply chain managers to manage variability propagation better and improve supply chain efficiency and carbon impact.

2. Related Literature

Our paper contributes to three streams of literature: upstream variability propagation/the bullwhip effect, inventory planning for perishables, and research on supply chain design and environmental impact.

Upstream Variability Propagation. There is an extensive body of research studying upstream variability propagation. This research, by and large, focuses on the variability amplification as it propagates upstream, i.e., the bullwhip effect, identifying its operational and behavioral causes (e.g., Lee et al., 1997; Moritz et al., 2021) and ways to alleviate them (e.g., Chen and Lee, 2009; Qu and Raff, 2021; Liu et al., 2022). Recent research empirically measures the degree of the upstream variability amplification (e.g. Cachon et al., 2007). The existing literature, however, does not consider product's perishability.

Our paper extends this literature by examining the degree of variability propagation upstream in the *perishable* product supply chains. In the absence of the factors that traditionally result in the upstream variability amplification, as expected, we observe *no* variability amplification for durable products. However, for perishable products, we find that both variability amplification/bullwhip effect and, surprisingly, variability dampening/anti-bullwhip effect could occur. The extent of such variability propagation upstream has a wide variation across different products, and is impacted by the market and product characteristics.

Inventory Management of Perishable Products. Numerous inventory management problems have been explored for durable products with varying characteristics and constraints (e.g., Ahn et al., 2021, Wei et al., 2021), while perishable inventory management remains a relatively understudied. We contribute to the sparse multi-echelon perishable inventory literature (see Kırcı et al., 2019, Mak et al., 2022 and references therein). We examine a multi-echelon perishable inventory model with endogenous demand formation for the upstream tier via optimal inventory replenishment of the downstream tier. To capture the optimal inventory replenishments of perishables, we build on Berk and Gürler (2008) and Belavina (2021). However, we additionally consider random product expiration time. Our paper explores perishable inventory management and its impact on variability propagation upstream and factors that impact variability propagation in the perishable product supply chains. To the best of our knowledge, our study is the first to provide such analysis, based on which we propose a new research direction for the contracts design/ supply chain coordination strategies via variability propagation control in perishables.

Supply Chain Design and Environmental Impact. Recent studies have expanded supply chain literature on the structure/contracts design (e.g. Su and Zhang, 2008; Ho et al., 2014) by exploring the environmental effects of various supply chain structures. For example, Park et al. (2015) and Dai and Tang (2022) explored carbon emission, supply chain design and regulation. Belavina (2021) explored the impact of grocery network design. Huang et al. (2022) and Manshadi and Rodilitz (2022) considered social responsibility, Lu et al. (2022) explored traceability and Zhang et al. (2022) investigated public safety. Our paper considers the impact of the supply chain variability propagation control on food waste and its environmental consequences. To the best of our knowledge, our study is the first to identify opportunities for creating coordinating mechanisms to improve the financial and environmental performance of supply chains, through better management of demand variability propagation.

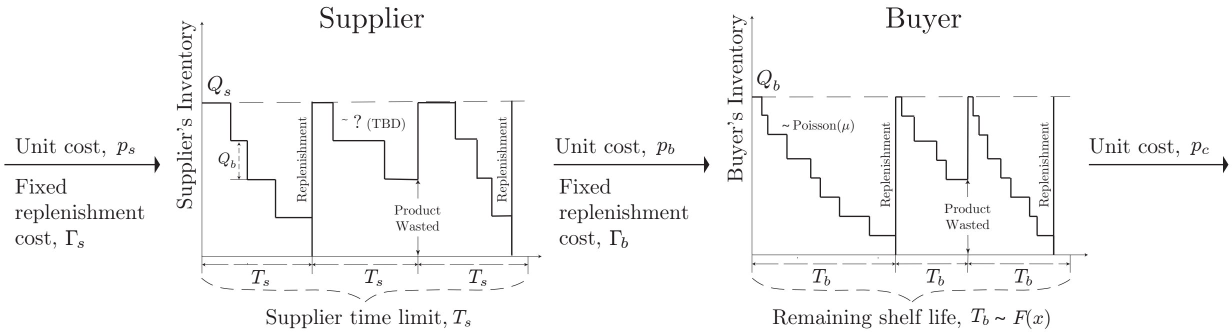


Figure 3.1. Model Setup.

3. Model Setup

Consider a two-echelon perishable product supply chain (see Figure 3.1).

The Buyer. To satisfy the continuing and uncertain customer demand, the buyer[1] buys the perishable product from the supplier and builds buyer inventories. Customer demand is generated according to a stochastic process with rate μ. Specifically, we assume that the customer demand arrives according to a Poisson process. The buyer procures the product at a unit price p_b and sells it to his customers at a price $p_c > p_b$. Each replenishment incurs a fixed cost Γ_b. We assume no lead time[2] between placing and receiving an order for the buyer. The buyer decides on the quantity to be procured Q_b to maximize his profit.

The Supplier. In turn, the supplier replenishes the perishable product and builds supplier inventories to meet demand from the buyer. The random demand that supplier faces is endogenously determined by the buyer's replenishment choices. We will discuss it in great detail in the next section. The supplier meets the demand by procuring the perishable product at a unit price p_s ($p_b > p_s$). Each replenishment incurs a fixed cost Γ_s. The supplier decides on the quantity to be procured Q_s to maximize his profit.

The Perishable Product. The perishable product has a random shelf life T. The supplier imposes a limit on how much time the perishable inventory can spend at the supplier's warehouse to guarantee certain minimum remaining shelf life at the time of shipping to their buyers (Kinarm, 2015; Fusin, 2017; Caemin Industries, 2018). We denote this time limit as T_s. Any product remaining in the supplier warehouse past T_s is discarded by the supplier, which generates supplier food waste. We

[1] For ease of exposition, we focus our main analysis on a setting with one buyer. Our findings remain intact in the multi-buyer case (see Section 9).

[2] Section 9 presents our analysis for a positive lead time scenario: our main findings continue to hold.

denote the remaining shelf life faced by the buyer as $T_b \equiv T - T_s$. Its resulting probability density and cumulative distribution functions are denoted by $f(\cdot)$ and $F(\cdot)$, respectively. All product unsold by the expiration time T_b perishes and has to be discarded by the buyer, which generates buyer food waste.

In sum, the buyer faces random demand generated by a stochastic process and manages his perishable inventory while knowing that the product will expire after a random number of days T_b. Notably, the purchase pattern of the buyer, resulting from the random buyer demand and the random product expiration time, leads to the endogenous formation of the random supplier demand. We discuss the endogenous formation of the random supplier demand in detail in the next section.

4. Supplier Demand

We next compute the mean and variance of the supplier demand based on the buyer's replenishment pattern, as each buyer's replenishment point generates an arrival of the supplier demand. In other words, the depletion process of the supplier's inventory is shaped by the length of each buyer's replenishment cycle. We denote buyer's replenishment cycle length by $CL_b(Q_b)$.

Proposition 1. *With buyer's replenishment quantity Q_b, the arrival rate of the supplier demand is $E[D_s] \approx \frac{Q_b}{E[CL_b(Q_b)]}$, and its variance is $var[D_s] \approx Q_b^2 \frac{var[CL_b(Q_b)]}{E[CL_b(Q_b)]^3}$.*
Here, the expected length of buyer's replenishment cycle is $E[CL_b(Q_b)] = \mu^{-1}(Q_b - \omega(Q_b))$, its variance is $var[CL_b(Q_b)] = \mu^{-2} E_{T_b}[Q_b(Q_b+1) - \sum_{k=0}^{Q_b}[Q_b(Q_b+1) - k(k-1)\psi_k(\mu T_b)] - (Q_b - \sum_{k=0}^{Q_b}(Q_b - k)\psi_k(\mu T_b))^2]$, with $\omega(Q_b)$ representing the expected number of the expired units, $\omega(Q_b) = E_{T_b}\left[\sum_{k=0}^{Q_b}(Q_b - k)\psi_k(\mu T_b)\right]$.

The buyer's order quantity Q_b determines the length of the buyer's replenishment cycle, and as a result, we find that it also impacts the supplier demand in the perishable product supply chains. The buyer's replenishments happen either from using up all stock (i.e., determined by the randomness of the demand) or from the product expiration (i.e., determined by the randomness of the expiration time T_b). That is, the buyer's *replenishment cycle length* differs from the buyer's *inventory depletion time*. Specifically, the inventory depletion time (the time of running out of all stock) is only determined by the buyer demand. While, the replenishment cycle length also takes into account the product expiration time, and is determined as the *minimum* of the *depletion time* and the *product expiration time*. Consequently, the expected length of each replenishment cycle is given by $E[CL_b(Q_b)] = \mu^{-1}(Q_b - \omega(Q_b))$: expected demand rate μ^{-1} times the expected number of units sold in a cylce $(Q_b - \omega(Q_b))$. Here, the term $\omega(Q_b)$ denotes the expected number of expired units. Thus, from Q_b units ordered by the buyer, only $Q_b - \omega(Q_b)$ units of inventory will be sold in expectation. The proof of Proposition 1 shows that the expected number of expired units $\omega(Q_b) = E_{T_b}\left[\sum_{k=0}^{Q_b}(Q_b - k)\psi_k(\mu T_b)\right]$, $\psi_k(\mu T_b) = \frac{(\mu T_b)^k}{k!}e^{-\mu T_b}$. That is, given a realization of the random buyer's expiration time T_b, $\psi_k(\mu T_b)$ is the probability of selling k units of product to his customers before the expiration. Thus, if the buyer sells $k \in [0, Q_b]$ units of product from his stock before expiration, the remaining $Q_b - k$ units of product will expire and generate waste. The expression for the variance of the buyer's replenishment cycle similarly follows.

We can now explore the extent of variability propagation upstream from the buyer to the supplier.

5. Extent of the Upstream Variability Propagation

The variability amplification ratio is commonly used to measure the extent of the upstream variability propagation (Cachon et al. 2007). It is the ratio of the coefficient of variation of the supplier and the buyer demands, $AR = CV_s/CV_b$. Proposition 1 allows us to obtain the coefficient of variation of the supplier demand:

$$CV_s(Q_b) \equiv \frac{\sqrt{var[D_s]}}{E[D_s]} = \sqrt{\frac{var[CL_b(Q_b)]}{E[CL_b(Q_b)]}}. \tag{5.1}$$

For the remainder of the paper, we use the term "variability of supplier demand" to represent its coefficient of variation. The above expression shows that the variability of supplier demand is determined by the *relative variance* (or variance-to-mean ratio) of the buyer's replenishment cycle length. For the remainder of the paper, we use the term "randomness" to represent the relative variance.

Equation (5.1) shows that the supplier demand variability $CV_s(Q_b)$ depends on the buyer order quantity. We next explore this relationship.

Theorem 1. *There exists a threshold $\bar{Q} \in [0, +\infty]$ such that $H(\bar{Q}) = 0$, and*

i. when the buyer's order quantity $Q_b \le \bar{Q}$, the variability of supplier demand is decreasing in Q_b, i.e., $\frac{d}{dQ_b}CV_s(Q_b) \le 0$;

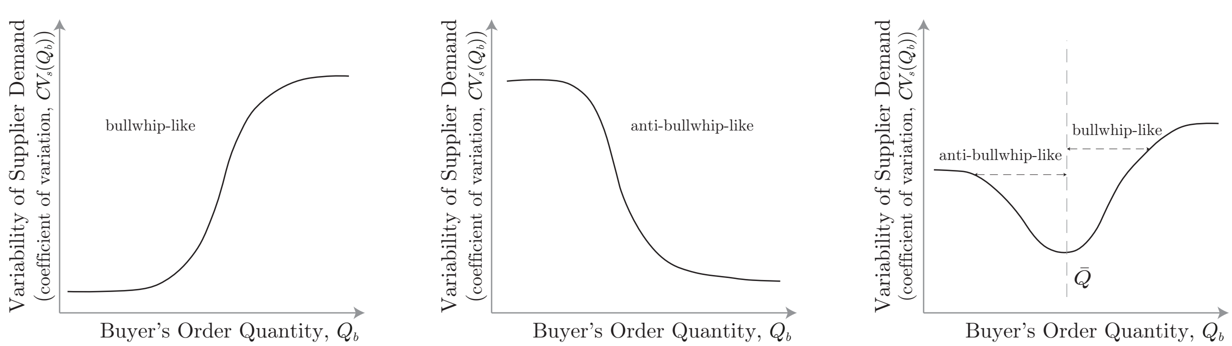
ii. when the buyer's order quantity $Q_b > \bar{Q}$, the variability of supplier demand is increasing in Q_b, i.e., $\frac{d}{dQ_b}CV_s(Q_b) \ge 0$, *where*

$$H(Q_b) = 2\mu^{-1}Q_b - \frac{1-\sum_{k=0}^{Q_b-1} E_{T_b}\left[\psi_k(\mu T_b)\right]}{1-\sum_{k=0}^{Q_b} E_{T_b}\left[\psi_k(\mu T_b)\right]} \left\{ \frac{E[CL_b(Q_b)^2]}{E[CL_b(Q_b)]} + E[CL_b(Q_b)] \right\}.$$

Theorem 1 shows that the extent of variability propagation up the supply chain depends in a nontrivial way on the order quantity of the buyer. As the coefficient of variation of the buyer demand does not change with buyer's order quantity ($CV_b = 1/\sqrt{\mu}$), the change in the amplification ratio is solely determined by the change in the supplier demand variability $CV_s(Q_b)$.

First, when the buyer's order quantity is lower than the threshold, the variability of supplier demand decreases as the buyer's order size increases. Whereas, when the buyer's order quantity is higher than the threshold, an increase in the buyer's order quantity leads to an increase in the supplier demand variability.

Overall, Theorem 1 establishes that there could exist three scenarios depending on the threshold $\bar{Q}$ (see Figure 5.1): as the buyer's order quantity increases, the amount of the variability that propagates up the supply chain is (1) increasing (i.e., $\bar{Q} = 0$), (2) decreasing (i.e., $\bar{Q} = +\infty$), or (3) first decreasing and then increasing (i.e., $0 < \bar{Q} < +\infty$). In the first scenario, a larger buyer's order quantity leads to more variability in supplier demand. That is to say, the downstream tier's purchasing pattern with a larger order quantity results in more variability propagation to the upstream tier. This is *akin* to the bullwhip effect, where variability propagates upstream in an amplified fashion. Hence, we refer to this amplification effect as the *bullwhip-like effect.* In the second scenario, the variability of upstream tier's demand decreases as the downstream tier's order size increases. It implies that the variability propagation up the supply chain reduces. In contrast to the first case, we refer to this effect as the *anti-bullwhip-like effect.* In the third scenario, a



(A) $\bar{Q} = 0$ (B) $\bar{Q} = +\infty$ (C) $0 < \bar{Q} < +\infty$

FIGURE 5.1. Extent of Variability Propagation.

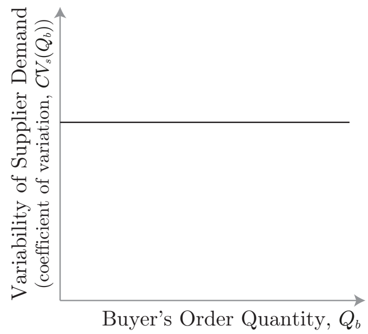


FIGURE 5.2. Variability Propagation in Durable Product Supply Chains.

larger order quantity first results in less variability propagation to the upstream tier and then in more variability propagation, i.e., the initial anti-bullwhip-like effect eventually transitions to the bullwhip-like effect.

It is worth mentioning that these two effects and the bullwhip effect concern the variability propagation in supply chains. However, the origins of the well-studied bullwhip effect and our (anti-) bullwhip-*like* effects are different. The bullwhip effect has been examined for the durable products and it stems from the non-stationarity of the downstream demand distribution (Lee et al., 1997). Whereas we study the perishable products in the absence of the demand non-stationarity, and we show that it is the product perishability that leads to bullwhip-like and anti-bullwhip-like effects. To illustrate this, we next consider the durable product version of our model, that is, a special case with the product shelf life $T = \infty$.

Proposition 2. *In the durable product supply chain, the variability of supplier demand is independent of the buyer's order quantity. Formally,* $CV_s(Q_b) \mid_{T=\infty} = 1/\sqrt{\mu}$, $\forall Q_b$.

Proposition 2 shows that the supplier sees no amplification of demand variability as it propagates from the buyer to the supplier (see Figure 5.2). That is, there is no bullwhip effect (i.e., $CV_s(Q_b) \mid_{T=\infty} = CV_b = 1/\sqrt{\mu}$ and, thus, amplification ratio $AR = 1$). This is in line with the bullwhip effect literature traditionally set in the durable product supply chains. As Lee et al. (1997) have shown, it is the non-stationarity of the downstream demand distribution that triggers the bullwhip effect. Here the buyer's demand distribution is stationary, and, as a result, no bullwhip-like effects arise when the product is durable. Interestingly, when perishable products are considered, even with a stationary downstream demand distribution, we find bullwhip-like and anti-bullwhip-like effects.

We next explain why the bullwhip-like and anti-bullwhip-like effects arise in perishable product supply chain, and describe when each of the three scenarios happens.

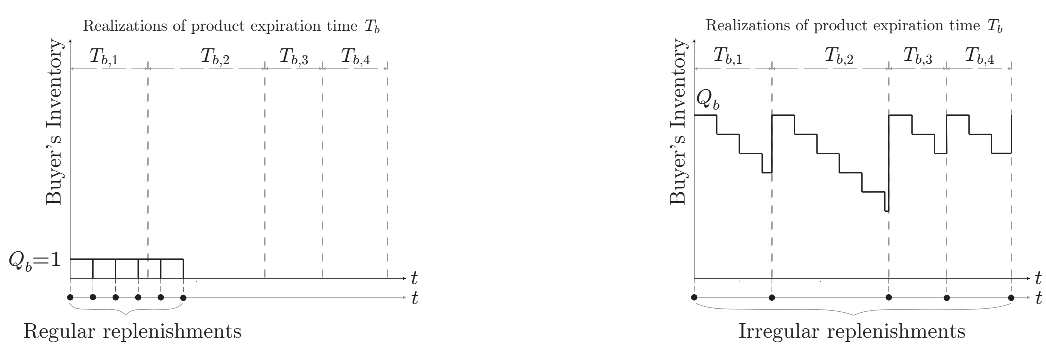
6. The Causes of Bullwhip-like and Anti-Bullwhip-like Effects

Recall that both the random buyer demand and random product expiration time contribute to the amount of variability in the supplier demand (via the buyer's replenishment cycle length). Thus, to identify the causes of distortion in variability propagation upstream in perishable product supply chains, we need to understand how the variability of both the buyer demand and the product expiration time translate into the variability of the supplier demand. We first identify the roles of these two random sources in the variability propagation by letting only one of them be random at a time. Then, we allow them both to be random simultaneously.

6.1. **Only Product Expiration Time is Random.** We first consider the setting with no variability in the buyer demand, while product expiration time is random.

Due to the product's perishable nature, the product expiration time sometimes truncates the buyer's inventory depletion process. To obtain an intuition of the variability propagation in this setting, consider two extreme cases: (1) the buyer purchases the minimum quantity of product from the supplier at each replenishment point ($Q_b = 1$), and (2) the buyer replenishes as many units as possible from the supplier in each cycle ($Q_b \to \infty$) (see Figure 6.1).

First, when the buyer orders only one unit of the product ($Q_b = 1$) at each replenishment (see Figure 6.1 (a)), his inventory level turns zero once a unit of its demand is realized. While it is possible that before the inventory level reaches zero, the random product expiration time might be



(A) Small Order Quantity ($Q_b = 1$).

(B) Large Order Quantity ($Q_b \to \infty$).

Figure 6.1. The Buyer's Replenishment Process (no demand variability).

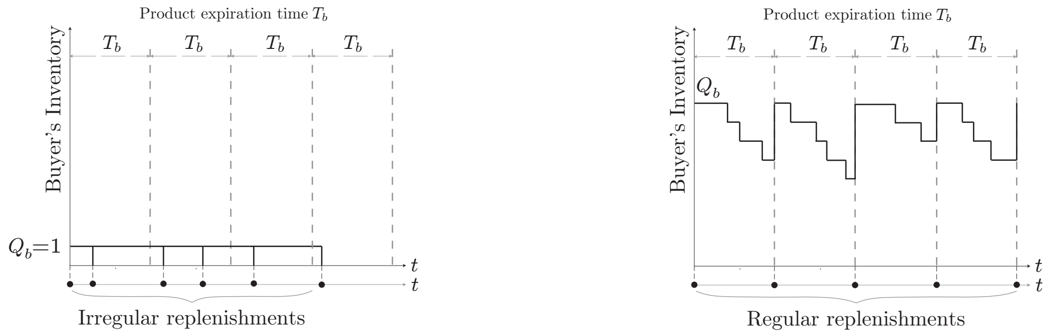
realized, this happens with a negligible probability due to the short inventory depletion time (under reasonable conditions, see Section 8). Whenever the inventory is depleted, the buyer restocks. That is, the buyer's replenishment process is nearly identical to the underlying arrival process of the buyer demand, with practically no impact of the product expiration time. Since, in this case, the buyer demand is steady and unvarying, the buyer's replenishment process will be practically as regular as the buyer's demand. This means practically no variability in the buyer's replenishment cycle length or, equivalently, in the supplier demand, see equation (5.1).

Second, suppose the buyer replenishes a substantial quantity of the product at each ordering point, namely, $Q_b \to \infty$ (see Figure 6.1 (b)). In this case, he holds a lot of inventory at the start of the cycle. As a result, due to the perishable nature of the product with a finite expiration time T_b, it is unlikely for the buyer to sell all the stock before the product expires. Consequently, the realized random product expiration time will always *truncate* the buyer's inventory depletion time. In other words, the buyer replenishes because of the product expiration rather than due to the inventory level dropping to zero. It follows that the buyer's replenishment process will be as variable as the product expiration time due to the truncations. Hence, compared to the first case, in the second case there is much more variability in the buyer's replenishment cycle length, and thus, in the supplier demand.

This observation can be generalized beyond these two extreme cases. The following theorem summarizes the variability propagation when there is no variability in the buyer demand and only the product expiration time is random.

Theorem 2. *When the buyer demand is deterministic and the product expiration time is random, the supplier demand variability is increasing in the buyer's order quantity, i.e.,* $\frac{d}{dQ_b}CV_s(Q_b) \geq 0$.

Theorem 2 reveals that with the deterministic buyer demand, the upstream demand variability increases as the order size of the downstream tier increases. So the bullwhip-like effect arises in this case: the variability propagation upstream from the buyer to the supplier increases. The root cause of the increase is that the truncation of the buyer's inventory depletion time by the random product expiration time becomes more likely to occur with the bigger buyer's order quantity.



(A) Small Order Quantity ($Q_b = 1$). (B) Large Order Quantity ($Q_b \to \infty$).

FIGURE 6.2. The Buyer's Replenishment Process (no expiration time variability).

6.2. **Only Buyer Demand is Random.** Now, we explore the setting in which the product expiration time is deterministic while the buyer demand is random. Similarly, consider the same two extreme cases (see Figure 6.2).

First, the buyer purchases the smallest quantity of product from the supplier during each replenishment cycle, namely, $Q_b = 1$ (see Figure 6.2 (a)). In this case, the buyer gets a replenishment as soon as one unit of customer demand arrives. Just like in Section 6.1, in this case, the buyer's replenishment process will look exactly like the underlying buyer demand. However, because now the buyer demand is random, the buyer's replenishments and, as a result, supplier demand will be as variable as the buyer's demand interarrival times.

Second, the buyer buys as many units as possible at each replenishment time, that is, $Q_b \to \infty$ (see Figure 6.2 (b)). The consequent high inventory level will take an extremely long time to end up at zero. Again, due to the perishable nature of the product (i.e., $T_b < +\infty$), the buyer's inventory depletion time will always be truncated by the deterministic product expiration time. Due to such truncation, the buyer's replenishment cycle length is exactly the same as the product expiration time T_b. Thus, there is no variability in the buyer's purchases (T_b is deterministic), and as a result, there is no variability in the supplier demand. Hence, compared to the first case, in the second case there is less variability in the buyer's replenishment cycle length, and thus, in the supplier demand.

The following theorem generalizes this beyond these two extreme cases and reveals how variability propagates upstream in this setting.

Theorem 3. *When the buyer demand is random and the product expiration time is deterministic, the supplier demand variability is decreasing in the buyer's order quantity, i.e.,* $\frac{d}{dQ_b} CV_s(Q_b) \leq 0$.

Theorem 3 shows that the anti-bullwhip-like effect occurs when the buyer demand is random and product expiration time is deterministic: the variability propagation up the supply chain reduces with an increase in buyer's order quantity. This is opposite to the setting when the buyer demand is deterministic and the product expiration time is random. Essentially, the root cause of the decrease is that the truncation of the buyer's demand depletion time by the deterministic product expiration becomes more likely to occur with the larger buyer's order quantity.

To summarize, in the two settings with just one random source considered in Sections 6.1 and 6.2, the buyer's order quantity plays an essential role in the variability propagation. With a larger buyer's order quantity, the buyer's inventory depletion time is more likely to be *truncated* by the product expiration time because the probability of selling all the existing inventory before expiration becomes more and more improbable. We refer to it as the *truncation effect.* On the one hand, if the product expiration time is random, a higher buyer's order quantity suggests that more randomness from the product expiration time will be picked up, which then translates into more variable supplier demand via this truncation effect. Hence, the larger the buyer's order size, the more the supplier demand variability will be. On the other hand, if the product expiration time is deterministic, with a higher buyer's order quantity, the random inventory depletion time will be more often truncated by the deterministic product expiration time. In other words, a higher buyer's order quantity captures more of the stability from the deterministic product expiration time through the truncation effect. Thus, an increase in the buyer's order quantity induces a decrease in the supplier demand variability.

We next combine these two settings: we allow both the buyer demand and product expiration time to be random and characterize when and which of the bullwhip-like and anti-bullwhip-like effects happen.

6.3. **Buyer Demand and Product Expiration Time are Random.** Consider a setting where both the buyer demand and product expiration time are random. Based on the truncation effect shown in Sections 6.1 and 6.2, it seems that with the random product expiration time, the above-referenced "stability" is removed, and we *should only see an increase* in the supplier demand variability with higher buyer's orders (i.e., the bullwhip-like effect). However, surprisingly, it turns out, that when both the buyer demand and the product expiration time are random, there exists a threshold of buyer's order quantity $\bar{Q}$ such that the extent of variability propagation is first surprisingly *decreasing* up to $\bar{Q}$ and only then increasing (see Theorem 1).

To explain this first-decreasing-then-increasing pattern of the upstream variability propagation, we first consider two edge cases. When the buyer purchases an extremely small quantity of product from the supplier during his replenishment cycle, the product expiration time nearly stands no chance of truncating the buyer's inventory depletion time. In this case, the buyer's replenishment process, and thus, the supplier demand, is solely determined by the underlying buyer demand. Therefore, in this case, the variability of supplier demand is expected to be only determined by the randomness of the buyer demand/inventory depletion time. On the flip side, when the buyer orders a sufficiently large quantity of product from the supplier, the product expiration time will be guaranteed to truncate the buyer's inventory depletion time. It follows that the variability of supplier demand should only be determined by the randomness of product expiration time in this case. Proposition 3 formalizes the relationship.

Proposition 3. ***Supplier Demand Variability at Extreme Buyer Order Quantities***

i. *When the buyer's order quantity is sufficiently small, the randomness (i.e., relative variance) of the buyer's inventory depletion time determines the variability of supplier demand,* $CV_s(Q_b \to 0^+) = \sqrt{1/\mu}$;
ii. *When the buyer's order quantity is sufficiently large, the randomness (i.e., relative variance) of the product expiration time* T_b *determines the variability of supplier demand,* $CV_s(Q_b \to \infty) = \sqrt{\sigma^2_{T_b}/\mu_{T_b}}$.

We next consider what happens beyond the above two edge cases.

As we saw in Sections 6.1 and 6.2, the buyer's inventory depletion time is occasionally truncated by the product expiration time (truncation effect). In the setting where both the buyer demand and product expiration time are random, this truncation by the expiration time induces the substitution from the randomness of the buyer's inventory depletion time to the randomness of product expiration time. Specifically, when buyer-inventory-depletion-time truncation by the product expiration time happens, at the expiration time, the remaining stock is discarded. As a result, instead of the randomness of buyer inventory depletion time, the randomness of product expiration time is responsible for the variability in supplier demand. So, when both the buyer demand and product

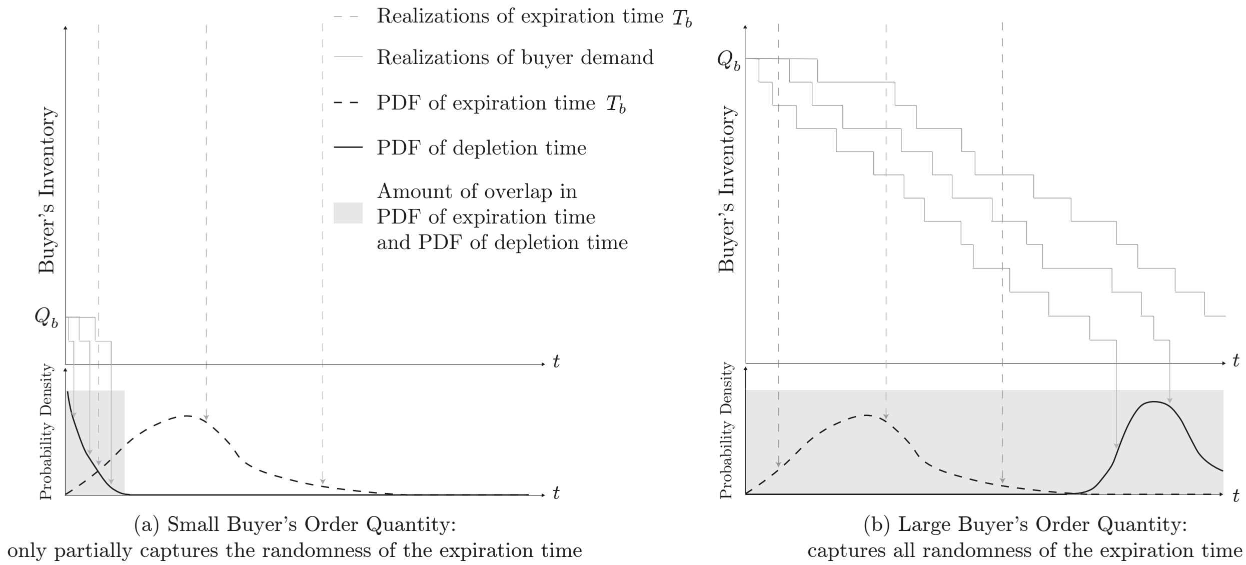


FIGURE 6.3. The Truncation Effect.

expiration time are random, the variability propagation upstream is more like a substitution (triggered by the truncation effect) from the randomness of inventory depletion time to the randomness of product expiration time. Figure 6.3 illustrates this substitution induced by the truncation effect. Panel (a) illustrates a case when buyer's order quantity Q_b is small, while panel (b) considers a larger order quantity. On both panels, dashed-gray lines depict a few (three) possible realizations of product expiration times, dashed-black line depicts the PDF of the product expiration time. Solid gray lines depict the buyer inventory depletion via (three) possible realizations of the buyer demand, and solid black line depicts the resulting PDF of the inventory depletion time.

When the buyer's order quantity Q_b is small (see Panel a), the depletion time of the buyer's inventory will be short. Such a short inventory depletion time will only be truncated by the expiration only when the realizations of the product expiration time are short. Conditional on truncation occurring, the relative variance of such product expiration times is low. For illustration, see the shaded area in Figure 6.3 (a) that shows the part of the PDF of expiration time T_b that is "activated" conditional on short inventory depletion times (due to small Q_b). That is, *when the buyer's order quantity Q_b is small, the conditional relative variance of product expiration times is low.*

When the buyer's order quantity Q_b is large, the realizations of the buyer's inventory depletion times are longer and, thus, can be truncated by more realizations of product expiration time (both short and long). Hence, conditional on the truncation occurring, the relative variance of the product expiration time is high. See the shaded area in Figure 6.3 (b) that shows the part of the PDF of expiration time T_b that is "activated" conditional on the longer inventory depletion times (due to large Q_b). That is, *when the buyer's order quantity Q_b is large, the conditional relative variance of product expiration times is high.*

Therefore, according to the truncation effect (a substitution from the buyer demand/inventory depletion time randomness to the product expiration time randomness), when the buyer's order quantity is small, the low conditional relative variance of product expiration times (as described above) will replace the randomness of buyer demand. As a result, this substitution induced by the truncation effect results in an initial decrease of variability propagation (i.e., we observe the anti-bullwhip-like effect). While, when the buyer's order quantity becomes large (see Figure 6.3 (b)), the conditional randomness of the product expiration time will eventually become higher than the randomness of buyer demand. That is, more variable product expiration time substitutes the buyer demand randomness, and passes on to the supplier demand. Hence, the extent of variability propagation eventually increases in the buyer's order quantity (i.e., we observe the bullwhip-like effect). Therefore, when both the buyer demand and the product expiration time are random, the anti-bullwhip-like effect eventually transitions to the bullwhip-like effect once the buyer order quantity surpasses the threshold $\bar{Q}$.

Now, we have established how the supplier demand variability changes as a function of the buyer's order quantity. To understand what kind of products or supply chains would be most affected by this, we need to explore how the supplier demand variability changes with market and product characteristics via buyer's optimal order size – this is the focus of our next section.

7. Impact of Market and Product Characteristics on the Extent of the Upstream Variability Propagation

We next examine how the extent of the upstream variability propagation is impacted by the market and product characteristics (product price, replenishment cost, mean and standard deviation of product expiration time and buyer demand). The variability of supplier demand is affected by randomness of the buyer demand/inventory depletion time and of the product expiration time. Since, the buyer's purchasing price and replenishment cost do not change the randomness of the buyer demand or the product expiration time, one would expect that buyer's purchasing price or replenishment cost would not have any effect on the variability of the supplier demand. One would also expect that increasing the mean or decreasing the standard deviation of the product expiration time would reduce randomness of expiration time, and consequently lower the variability of the supplier demand. Similarly, one would expect that increasing the buyer demand rate would lower randomness of buyer demand and, as a result, the variability of the supplier demand. We indeed find that the variability of supplier demand is decreasing in the buyer demand rate μ.[3] However, we find an unexpected impact of changes in the other market and product characteristics. Specifically,

Result 1: *The variability of supplier demand is first decreasing and then increasing in the buyer's purchasing price (p_b), the buyer's replenishment cost (Γ_b), the mean and standard deviation of product expiration time (μ_{T_b} and σ_{T_b}).*

That is, instead of the expected no impact, we find a first-decreasing-then-increasing impact of an increase in the buyer's purchasing price p_b and the buyer's replenishment cost Γ_b on the supplier demand variability. Moreover, rather than the expected monotonic decrease in the mean of product expiration time (μ_{T_b}), there is an initially expected decrease in the supplier demand variability followed by an unexpected increase. Further, instead of the expected monotonic increase in the standard deviation of product expiration time (σ_{T_b}), we observe first a surprising decrease in the supplier demand variability followed by the expected increase. We next uncover why these unexpected effects occur.

For any market and product characteristic $\mathcal{X}$, its effect on upstream variability propagation can be decomposed into two components:

$$\frac{dCV_s(Q_b)}{d\mathcal{X}} |_{Q_b=Q_b^*} = \frac{\partial CV_s(Q_b)}{\partial \mathcal{X}} |_{Q_b=Q_b^*} + \frac{dCV_s(Q_b)}{dQ_b} |_{Q_b=Q_b^*} \cdot \frac{dQ_b^*}{d\mathcal{X}}. \tag{7.1}$$

The first term captures the direct effect, while the second term captures its indirect effect through the change in the buyer's optimal order quantity Q_b^* (Appendix, Lemma 2 provides the derivation of Q_b^*). The combination of the direct and indirect effects makes the prediction of the overall effect on the change in the supplier demand variability more complicated, yet it explains the unexpected results.

In Theorem 1, we established how the change in buyer's ordering quantity impacts the extent of variability propagation, that is, we know how the term $\frac{dCV_s(Q_b)}{dQ_b}$ behaves. Thus, it remains to account

[3]Note that the mean and standard deviation of the buyer demand are linked for the Poisson process. We will decouple the two via normal approximation, and explore the impact of the two separately in section 7.5.

for the direct effect $\frac{\partial CV_s(Q_b)}{\partial \mathcal{X}}$ and, within the indirect effect, for the impact on buyer's optimal order quantity $\frac{dQ_b^*}{d\mathcal{X}}$.

We next explore the impact of the following characteristics on the supplier demand variability: the buyer's purchasing price p_b, the buyer's replenishment cost Γ_b, the mean and standard deviation of product expiration time μ_{T_b} and σ_{T_b}, and the buyer demand rate μ. For each characteristic χ, we proceed in the following three steps: (1) we investigate the direct effect $\frac{\partial CV_s(Q_b)}{\partial \mathcal{X}}$, that is, how the curve of $CV_s(Q_b)$ shifts with χ; (2) we examine the effect of χ on the buyer's optimal order quantity $\frac{dQ_b^*}{d\mathcal{X}}$; and (3) we combine the direct and indirect effects to understand the overall impact of χ on the supplier demand variability $\frac{dCV_s(Q_b)}{d\mathcal{X}}|_{Q_b=Q_b^*}$.

7.1. Impact of Price and Replenishment Cost.

We start with the supplier's selling price/buyer's purchasing price p_b and buyer's replenishment cost Γ_b.

Direct Effect. The price p_b and the cost Γ_b that buyer pays do not directly impact supplier's demand variability $\frac{\partial CV_s(Q_b)}{\partial p_b} = \frac{\partial CV_s(Q_b)}{\partial \Gamma_b} = 0$ (see equation 5.1).

Effect on Buyer's Optimal Quantity. The price p_b and the replenishment cost Γ_b that buyer pays do impact the buyer's optimal order quantity.

Lemma 1. *The buyer's optimal order quantity Q_b^* is (i) decreasing in the purchasing price p_b, $\frac{dQ_b^*}{dp_b} \leq 0$, and (ii) increasing in the replenishment cost Γ_b, $\frac{dQ_b^*}{d\Gamma_b} \geq 0$.*

The effects of the purchasing price p_b and the replenishment cost Γ_b on the buyer's optimal order quantity Q_b^* (Lemma 1) follow directly from the tradeoff that determines buyer's optimal order quantity: procuring too many items results in more chances of being unable to sell all the product before expiration and thus in more waste, while buying too few items leads to more frequent purchases and triggers additional replenishment costs. Thus, as the purchasing price p_b increases, wasting becomes more costly and the buyer is less willing to procure more products at a time and the optimal order quantity decreases. In the same manner, if the replenishment cost for each order is higher, frequent purchases will be less favorable. As a result, the buyer will procure more of the product at each ordering point to avoid the higher fixed cost.

Combining Direct and Indirect Effects. Since, there is no direct effect: $CV_s(Q_b)$ curve does not move with change in p_b or Γ_b, we only need to combine the impact of change in order quantity

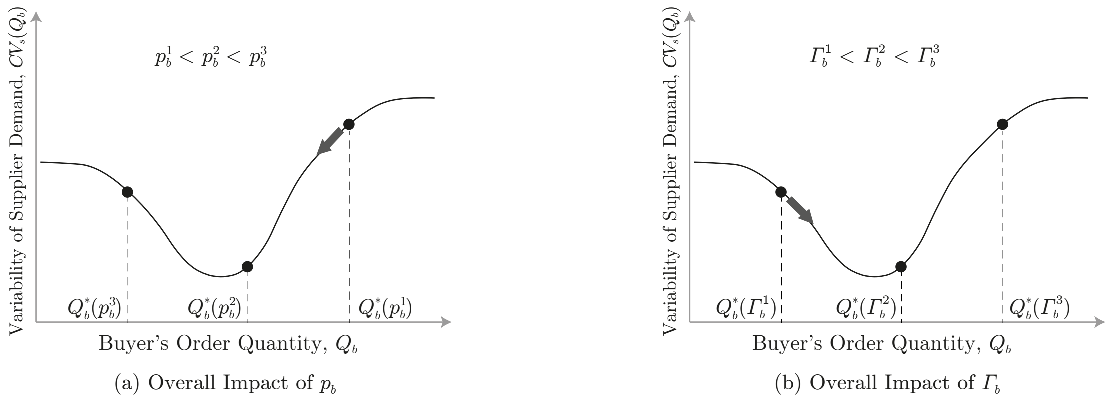


FIGURE 7.1. The Impact of Buyer's Purchasing Price p_b and Replenishm. Cost Γ_b.

on variability propagation from Theorem 1 and the impacts of the buyer's purchasing price p_b and replenishment cost Γ_b on the buyer's optimal order quantity from Lemma 1. The buyer's optimal order quantity is a monotonic function of both p_b and Γ_b (Lemma 1). As Figure 7.1 demonstrates, with the change in either, the optimal order quantity will move along the $CV_s(Q_b)$ curve (replicated from Figure 5.1 (c)), tracing its first-decreasing-then-increasing pattern. In a word, with change in p_b and Γ_b, it is the consequent monotonic change in the buyer's optimal order quantity that induces the first-decreasing-then-increasing pattern in the variability propagation change.

Interestingly, the origins of the first-decreasing-then-increasing pattern associated with an increase in the mean and standard deviation of the product expiration time are entirely different from those of price and replenishment cost. The effect of the mean and standard deviation of the product expiration time (and the buyer demand rate) are more involved, as they directly affect the variability of supplier demand, i.e., $\frac{\partial CV_s(Q_b)}{\partial \mathcal{X}} \neq 0, \mathcal{X} \in \{\mu_{T_b}, \sigma_{T_b}, \mu\}$. That is, as these parameters change, the $CV_s(Q_b)$ curve will shift. Due to the complex nature of the interaction between the randomness of the buyer demand and randomness of the expiration time, it is hard to analytically characterize the direct and indirect effects. Luckily, we are able to show that for all plausible parameters (see section 8) the following results hold.

7.2. **Impact of the Mean of the Product Expiration Time.** As one might expect, increasing the mean of the product expiration time μ_{T_b} reduces the randomness of expiration time, contributing to a decrease in the supplier demand variability. However, we find that an initial decrease in the supplier demand variability is, surprisingly, followed by an *increase*. We next explain this

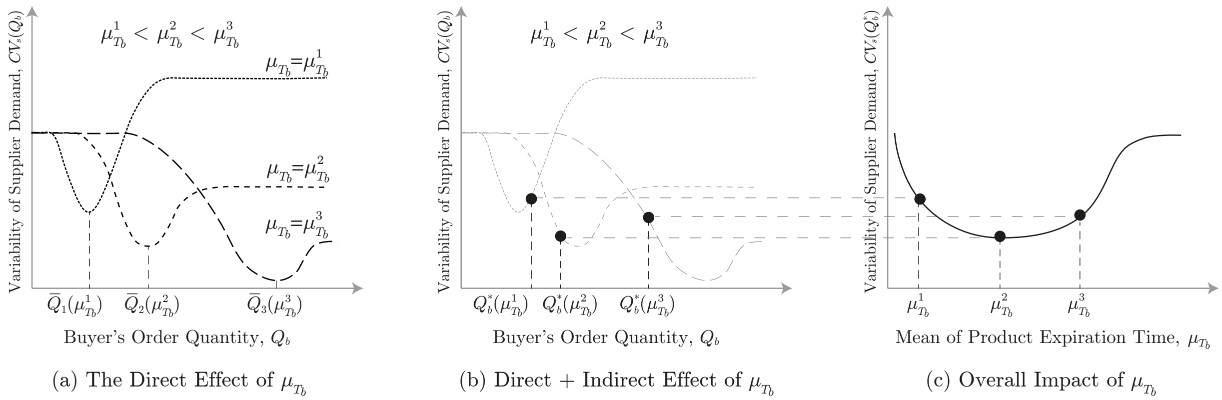


FIGURE 7.2. The Impact of the Mean of Product Expiration Time μ_{T_b}.

unexpected first-decreasing-then-increasing pattern resulting from the increase in the mean of the product expiration time.

The mean of the product expiration time μ_{T_b} gives rise to both the direct and indirect effects on the supplier demand variability. We first explore its direct effect.

Direct Effect. The direct effect $\frac{\partial CV_s(Q_b)}{\partial \mu_{T_b}}$ measures how the mean of the product expiration time μ_{T_b} shifts the $CV_s(Q_b)$ curve. As depicted by Figure 7.2 (a), the curve of $CV_s(Q_b)$ shifts downward and right when μ_{T_b} increases. Such movement results from the following: with a *larger mean of the product expiration time* μ_{T_b}, (1) the point of origin stays the same – this is determined by the randomness of buyer's inventory depletion time $1/\sqrt{\mu}$ (Proposition 3) and is independent of μ_{T_b}; (2) the point of convergence on the right is lower – this is determined by the randomness of product expiration time $\sqrt{\sigma^2_{T_b}/\mu_{T_b}}$ (Proposition 3), which reduces with larger μ_{T_b}; (3) the curve of $CV_s(Q_b)$ starts to drop at a higher order quantity – due to a later kick-in of the truncation effect (the substitution from randomness of buyer demand to randomness of product expiration time) (Figure 6.3); (4) the threshold $\bar{Q}$, that determines the turning (lowest) point on the graph (Theorem 1), is higher and the variability of the supplier demand at $\bar{Q}$, $CV_s(\bar{Q})$, is lower – due to the lower randomness of product expiration time; and (5) the curve of $CV_s(Q_b)$ is wider and more flat – due to a slower speed of substitution (from the truncation effect) resulting from lower randomness of product expiration time.

Overall, as μ_{T_b} increases, the randomness of the product expiration time decreases, so less variability passes on to the supplier and the truncation effect kicks in later, and ultimately the curve of $CV_s(Q_b)$ becomes lower and moves toward the right side (Figure 7.2 (a)).

Effect on Buyer's Optimal Quantity. We now discuss how the mean of the product expiration time impacts the buyer's optimal order quantity $\frac{dQ_b^*}{d\mu_{T_b}}$.

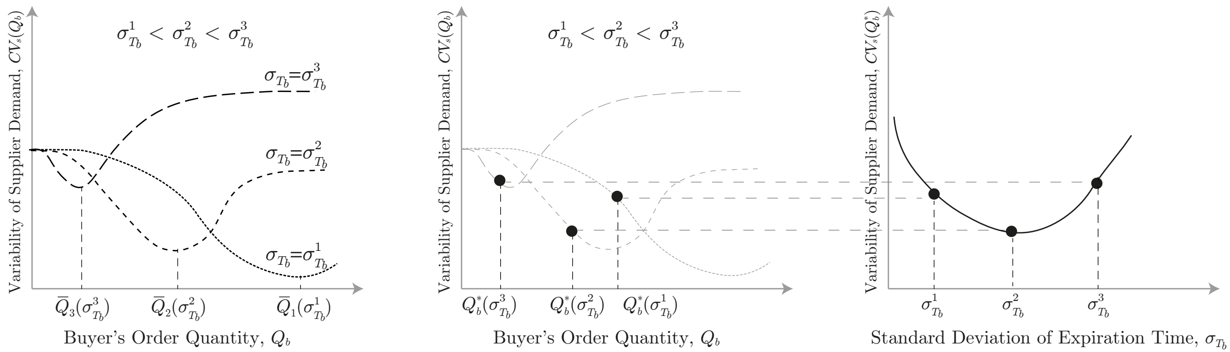
***Observation 1*:** *The buyer's optimal order quantity Q_b^* is increasing in the mean of the product expiration time μ_{T_b}.*

The buyer's optimal order quantity is determined by a tradeoff between the product waste cost from buying too much and the additional replenishment cost from buying too little. Intuitively, with a larger mean of the product expiration time μ_{T_b}, the product in expectation will take a longer time to expire. Thus, the buyer can buy more products in each ordering cycle and save on the replenishment cost without generating additional product waste.

Combining Direct and Indirect Effects. The direct effect moves the curve of $CV_s(Q_b)$ downward and right, and the indirect effect leads to an increase in the buyer's optimal order quantity. Figure 7.2 (b) overlays the optimal order quantity change with the change in the mean of product expiration time μ_{T_b} over Figure 7.2 (a), which allows us to map out Figure 7.2 (c). Figure 7.2 (c) shows how the curve $CV_s(Q_b^*)$, which depicts the variability of the supplier demand at the buyer's optimal order quantity Q_b^*, looks as a function of the mean of product expiration time μ_{T_b}. When the mean of the product expiration time μ_{T_b} increases, the curve $CV_s(Q_b)$ shifts down and right and its corresponding optimal order quantity Q_b^* shifts right (see Figure 7.2 b). At first, the downward movement of the curve dominates the right shift of the optimal order quantity Q_b^*, resulting in a decreasing pattern of $CV_s(Q_b^*)$, see Figure 7.2 (c). However, eventually, the faster rightward movement of $CV_s(Q_b)$ dominates the slower rightward movement of the optimal order quantity and leads to an increase in the supplier demand variability $CV_s(Q_b^*)$.

Note that for the product expiration time μ_{T_b}, as opposed to the case of price p_b and replenishment cost Γ_b, the first-decreasing-then-increasing pattern of the curve $CV_s(Q_b)$ does not determine the first-decreasing-then-increasing pattern of $CV_s(Q_b^*)$. It is the slower buyer's optimal quantity Q_b^* response to the increase of μ_{T_b} (slower right-shift) as compared to the response of $CV_s(Q_b)$ (faster right-shift), that generates an ultimate unexpected increase in the supplier demand variability.

7.3. **Impact of the Standard Deviation of the Product Expiration Time.** One might expect that increase in the standard deviation of the product expiration time σ_{T_b}, i.e., increase in the randomness of the product expiration time, would result in more variability passing on to the



(a) The Direct Effect of σ_{T_b} (b) Direct + Indirect Effect of σ_{T_b} (c) Overall Impact of σ_{T_b}

FIGURE 7.3. The Impact of the St. Deviation of Product Expiration Time σ_{T_b}.

supplier and, thus, higher supplier demand variability. However, with the increase in the standard deviation of the product expiration time σ_{T_b} we observe first a surprising decrease in the supplier demand variability followed by the expected increase.

To understand the reasons for the existence of the unexpected decreasing pattern with an increase in the standard deviation, we first consider the direct effect.

Direct Effect. When the standard deviation σ_{T_b} becomes larger, the product expiration time becomes more variable, and its distribution is wider and more flat. Further, we find that the curve of $CV_s(Q_b)$ moves upward and left (see Figure 7.3 a). Such an upward and left movement is based on the following: with a *higher standard deviation of product expiration time* σ_{T_b}, (1) the point of origin stays the same – this is determined by the randomness of buyer's inventory depletion time $1/\sqrt{\mu}$ (Proposition 3) and is independent of σ_{T_b}; (2) the point of convergence on the right is higher – this is determined by the randomness of the product expiration time $\sqrt{\sigma^2_{T_b}/\mu_{T_b}}$ (Proposition 3), which increases with higher σ_{T_b}; (3) the curve of $CV_s(Q_b)$ starts to drop at a lower order quantity – due to an earlier kick-in of the truncation effect (the substitution from randomness of buyer demand to randomness of product expiration time) (Figure 6.3); (4) the threshold $\bar{Q}$, that determines the turning (lowest) point on the graph (Theorem 1), is lower and the variability of the supplier demand at $\bar{Q}$, $CV_s(\bar{Q})$, is higher – due to the higher randomness of product expiration time; and (5) the curve of $CV_s(Q_b)$ is narrower and less flat – due to a faster speed of substitution (from the truncation effect) resulting from the higher randomness of product expiration time.

Altogether, as σ_{T_b} increases, the randomness of the product expiration time increases, so more variability passes on to the supplier and the truncation effect kicks in earlier, and ultimately the curve of $CV_s(Q_b)$ becomes higher and moves toward the left side as depicted on Figure 7.3 (a).

Effect on Buyer's Optimal Quantity. The impact of the standard deviation of product expiration time on the buyer's optimal order quantity $\frac{dQ_b^*}{d\sigma_{T_b}}$ depends on the relative magnitude of the buyer's purchasing price p_b and replenishment cost Γ_b. This is akin to the result seen in the newsvendor model: with higher demand variability, optimal order quantity would go further away from the mean either above (if cost of underage is greater than the cost of overage) or below (otherwise) (Arrow et al., 1951). In our setting, roughly speaking, the purchasing price p_b plays the role of overage and the replenishment cost Γ_b plays the role of underage. When the replenishment cost Γ_b is drastically higher than the purchasing cost p_b, the buyer's optimal order quantity would be increasing in the standard deviation of product expiration time σ_{T_b}. Otherwise, the buyer's optimal order quantity is decreasing in the standard deviation of product expiration time σ_{T_b}. We find that for all plausible parameters (see section 8) the latter holds and the buyer's optimal order quantity Q_b^* is *decreasing* in the standard deviation of product expiration time σ_{T_b}:

Observation 2: *The buyer's optimal order quantity Q_b^* is decreasing in the standard deviation of the product expiration time σ_{T_b}.*

Combining Direct and Indirect Effects. The direct effect moves the curve of $CV_s(Q_b)$ upward and left, and the indirect effect leads to a monotonic decrease in the buyer's optimal order quantity. Figure 7.3 (b) overlays the optimal order quantity change with the change in the standard deviation of product expiration time σ_{T_b} over Figure 7.3 (a), which allows us to map out Figure 7.3 (c). It combines the direct and indirect effects, and shows how the variability of the supplier demand $CV_s(Q_b^*)$ at the buyer's optimal order quantity Q_b^*, looks like a function of the standard deviation of product expiration time σ_{T_b}. As the standard deviation of product expiration time σ_{T_b} increases, initially an earlier drop of the curve $CV_s(Q_b)$, because of an earlier kick-in of truncation effect (due to wider distribution of the product expiration time and the consequent more realizations of shorter expiration time), results in a decreasing pattern of $CV_s(Q_b^*)$. However, eventually the faster leftward and up movement of the $CV_s(Q_b)$ curve outpaces the slower leftward movement of the optimal order quantity and leads to an eventual increase in the supplier demand variability $CV_s(Q_b^*)$.

Again, the first-decreasing-then-increasing pattern of the curve $CV_s(Q_b)$ does not determine the first-decreasing-then-increasing pattern of $CV_s(Q_b^*)$. It is the earlier kick-in of the truncation effect response to the increase of σ_{T_b} that leads to an initial unexpected decrease in the supplier demand variability.

7.4. **Impact of the Buyer Demand Rate.** Intuitively one would expect that as the buyer demand rate μ increases, the randomness of buyer demand will decrease and less variability will propagate upstream, resulting in a lower supplier demand variability. Our findings validate this intuition. We find that despite the existing first-decreasing-then-increasing pattern in the curve of $CV_s(Q_b)$, the variability of supplier demand is decreasing in the buyer demand rate μ. As opposed to the contrasting forces existing in the effect of product characteristics μ_{T_b} and σ_{T_b} above, with change in μ, both direct and indirect effects work in the same direction, and thus, no unexpected impact of the buyer demand rate μ arises (interested reader can find the detailed analysis in Appendix C).

7.5. **Impact of the Mean and Standard Deviation of the Buyer Demand under Normal Approximation.** In the Poisson arrival process, the mean and variance of the buyer demand are coupled. We show, that even when these two demand characteristics are decoupled (through the normal approximation of the Poisson distribution), the effect of the mean of the buyer demand is the same as described in Section 7.4.

We next describe the impact of the change in the standard deviation. Again, one would expect that an increase in the standard deviation of the buyer demand σ_d, i.e., increase in the randomness of the buyer demand, would result in more variability passing on to the supplier and, thus, higher supplier demand variability. However, with the increase in the standard deviation of the buyer demand σ_d we observe first a surprising decrease in the supplier demand variability followed by the expected increase.

Result 2: *The variability of supplier demand is first decreasing and then increasing in the standard deviation of buyer demand σ_d.*

In contrast to the in-line-with-expectation-impact of the mean of the buyer demand, we find the unexpected first-decreasing-then-increasing pattern of the standard deviation of buyer demand σ_d. While the effect of the increase in the standard deviation of buyer demand σ_d is quite elaborate (see Appendix D), the origin of the initial (unexpected) decrease is similar to that of the standard deviation of the product expiration time σ_{T_b}. In short, it is the earlier kick-in of the truncation effect, resulting from the *wider* distribution of the buyer demand and its consequent more realizations of shorter depletion time (i.e., more instances of truncation happening at lower order quantity) in

	p_b (\$/lb)	*Margin* (%)	p_c (\$/lb)	Γ_b (\$/replen.)	μ_{T_b} (days)	$\sqrt{3}\sigma_{T_b}$ (days)	μ_d (lbs/year)	σ_d (lbs/year)
Orange Juice	1.50	45.5	2.75	$13.5 + 0.51x$	10.5	3.5	$16nz$	$5.92\sqrt{nz}$
Bread	0.94	41	1.6	$9 + 0.45x$	6	1	$53nz$	$2.87\sqrt{nz}$
Milk	0.36	30	0.51	$13.5 + 0.51x$	8.5	1.5	$141nz$	$15.36\sqrt{nz}$
Cheddar	2.20	60	5.49	$13.5 + 0.51x$	24.5	3.5	$10.1nz$	$0.35\sqrt{nz}$
Chicken	0.86	50	1.72	$13.5 + 0.51x$	1.5	0.5	$95.8nz$	$5.83\sqrt{nz}$
Ground beef	2.32	50	4.63	$13.5 + 0.51x$	1.5	0.5	$58.4nz$	$3.72\sqrt{nz}$
Steak	5.33	50	10.66	$13.5 + 0.51x$	1.5	0.5	$58.4nz$	$3.72\sqrt{nz}$
Lettuce	2.23	30	3.18	$13.5 + 0.51x$	10	1	$11.7nz$	$1.6\sqrt{nz}$
Tomatoes	1.46	20	1.83	$13.5 + 0.51x$	7	1	$18.2nz$	$0.87\sqrt{nz}$
Cucumbers	0.64	48.8	1.26	$13.5 + 0.51x$	8.5	1.5	$7.5nz$	$0.63\sqrt{nz}$
Onions	1.43	20	1.79	$9 + 0.45x$	35	7	$20.4nz$	$1.95\sqrt{nz}$
Potatoes	0.66	20	0.83	$9 + 0.45x$	28	7	$30.3nz$	$2.91\sqrt{nz}$
Carrots	0.49	48.8	0.95	$9 + 0.45x$	24.5	3.5	$8.3nz$	$1.72\sqrt{nz}$
Bananas	0.54	15	0.64	$13.5 + 0.51x$	4.5	2.5	$27.2nz$	$1.65\sqrt{nz}$
Strawberries	1.22	55	2.71	$13.5 + 0.51x$	6	1	$8.5nz$	$0.73\sqrt{nz}$

Note: (1) The buyer's purchasing price p_b = (1 - Margin%) $\times$ selling price p_c (see Statista, 2017; Laurel Grocery Company, 2018; Busby, 2020; Deese et al., 2021; Naveo Commerce, 2021; MoneyPluck, 2022 for the margin and U.S. Bureau of Labor Statistics, 2022 for p_c). (2) The replenishment fixed cost $\Gamma_b = a + 2bx$, where a denotes fixed cost of a reefer (\$13.5) or of a dry van (\$9) for each purchase (see ABCO Transportation, 2016), the term b denotes fuel cost of a reefer (\$0.51) or of a dry van (\$0.45) per mile (see ATBS, 2021), and the total fuel costs $2bx$ are proportional to the travel distance between the supplier and the buyer $x \in [5, 800]$ miles (see King et al., 2010; Consumer Ecology, 2022). (3) The product expiration time's mean μ_{T_b} and standard deviation σ_{T_b} are obtained from Eat By Date (2022). (4) The buyer demand mean $\mu_d \approx$ mean of daily consumptions/person (see Statista, 2021) $\times$ nz, n =7 days/wk / 1.5 trips/wk (see Mitova, 2021), daily number of customers served $z \in [450, 4000]$ (see Kroger, 2017; IBISWorld, 2021; Mitova, 2021; Statista, 2021). (5) The buyer demand standard deviation σ_d $\approx$standard deviation of daily consumptions/person (see Statista, 2021) $\times$ $\sqrt{nz}$.

TABLE 2. Market and Product Characteristics.

response to the increase of σ_d, that leads to an initial unexpected decrease in the supplier demand variability.

Knowing how these market and product characteristics change the extent of variability propagation, we can derive their implications for various business conditions. That is, the observations above help reveal what kind of products/supply chains are most affected by the variability propagation in perishable product supply chains. We will discuss this in the next section.

8. Model Calibration

We calibrate our model using data from various industry reports and academic studies. To account for the wide heterogeneity in the market and product characteristics, table 2 provides the parameter estimates for a number of perishable products.

8.1. **Variability Amplification Ratio.** We first examine how the variability propagation differs across various perishable products. Figure 8.1 illustrates for various products the variability am-

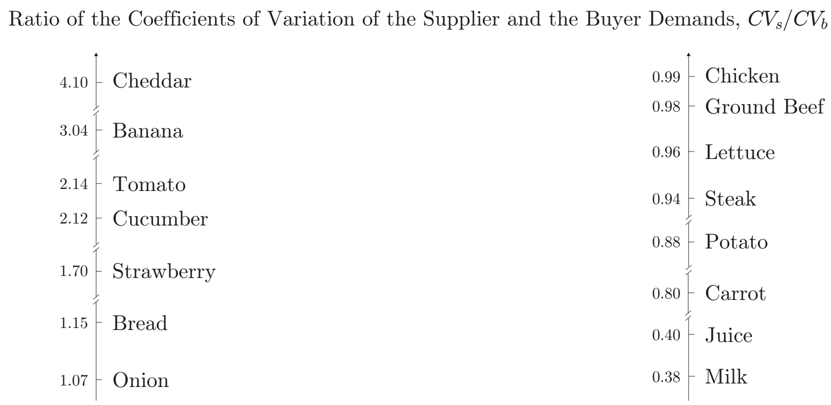


FIGURE 8.1. The Amplification Ratio, $AR = CV_s/CV_b$.

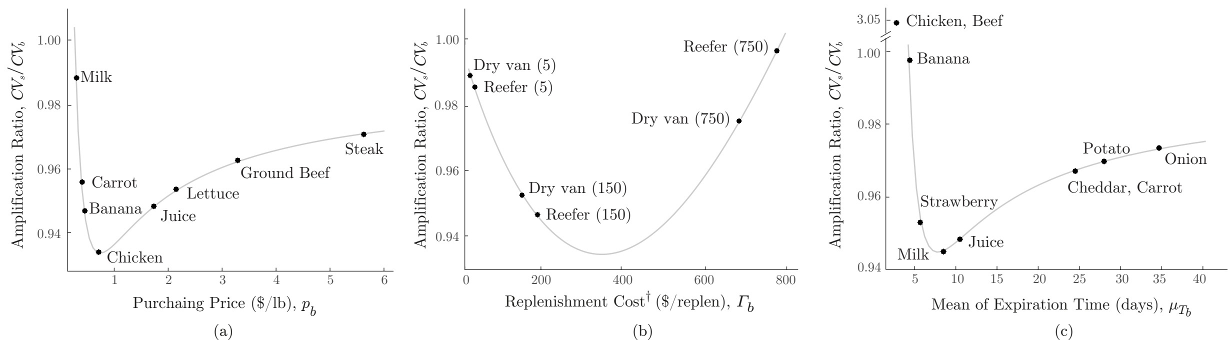
plification ratio. We display the products with an amplification ratio greater than one ($AR > 1$) on the left and those with a ratio less than one ($AR < 1$) on the right. Recall, when the ratio is greater than one, bullwhip effect is said to occur (Cachon et al. 2007). That is, we find that for some products the bullwhip effect happens and for others anti-bullwhip effect occurs.

Figure 8.1 further demonstrates that the degree of variability propagation/amplification varies considerably across products. The differences result from more/less favorable combinations of the product characteristics. Some combinations result in high variability amplification, while others substantially dampen variability propagation. We next examine the degree of the impact of each of the characteristics separately. To do so, we vary *one* characteristic at a time, and set the other characteristics to the baseline level[4].

Figure 8.2 illustrates the impact of purchasing price, replenishment cost and mean of the product expiration time on the amplification ratio. The change in the amplification ratio is determined by the change in the supplier demand variability CV_s as buyer's demand variability CV_b is unaffected by these parameters. Figure 8.2 shows that the amplification ratio is indeed decreasing first and then increasing in the buyer's purchasing price p_b, replenishment cost Γ_b and the mean of product expiration time μ_{T_b} mimicking the change in the supplier demand variability (as discussed in section 7).

Figure 8.3 shows the impact on the amplification ratio of the mean and standard deviation of the buyer demand, and that of the standard deviation of the product expiration time. In line with our analysis in section 7, we observe that the supplier demand variability CV_s is indeed decreasing

[4]We use as the baseline the parameters for lettuce (Table 2) with $x = 150$ and $z = 1000$.

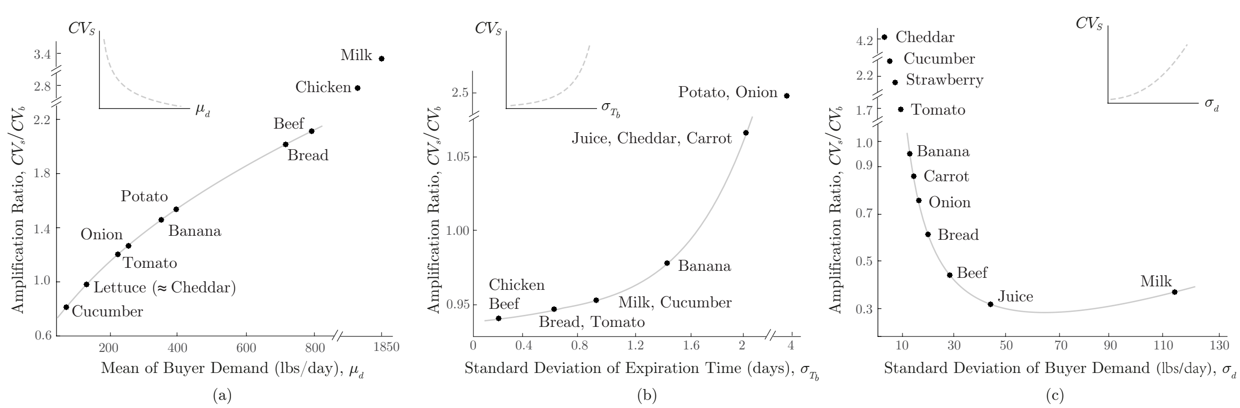


Note: Drawn for the baseline value of all parameters except for the parameter varied on x-axis. †The travel distance x (in miles) is shown in parentheses.

FIGURE 8.2. Impact on the Amplification Ratio CV_s/CV_b.

with an increase of mean buyer demand, μ_d (see the callout in Panel a). However, despite the decrease in CV_s, the amplification ratio CV_s/CV_b is increasing with higher mean buyer demand (see Panel a). Higher mean buyer demand also reduces the buyer demand variability CV_b. This direct reduction in CV_b with the increase in the mean of buyer demand overtakes its indirect reduction in supplier demand variability CV_s, resulting in the increase of the amplification ratio.

The change in amplification ratio with the change in the standard deviation of product expiration time σ_{T_b} (see Panel b) is again determined by the change in the supplier demand variability CV_s as the buyer's demand variability CV_b is unaffected by it. Our investigation shows that in local markets (i.e., those with shorter travel distances) and/or for expensive perishable products, as predicted in section 7, the supplier demand variability CV_s is first decreasing and then increasing in σ_{T_b}. However, for the baseline parameters where the replenishment travel distances are substantial,



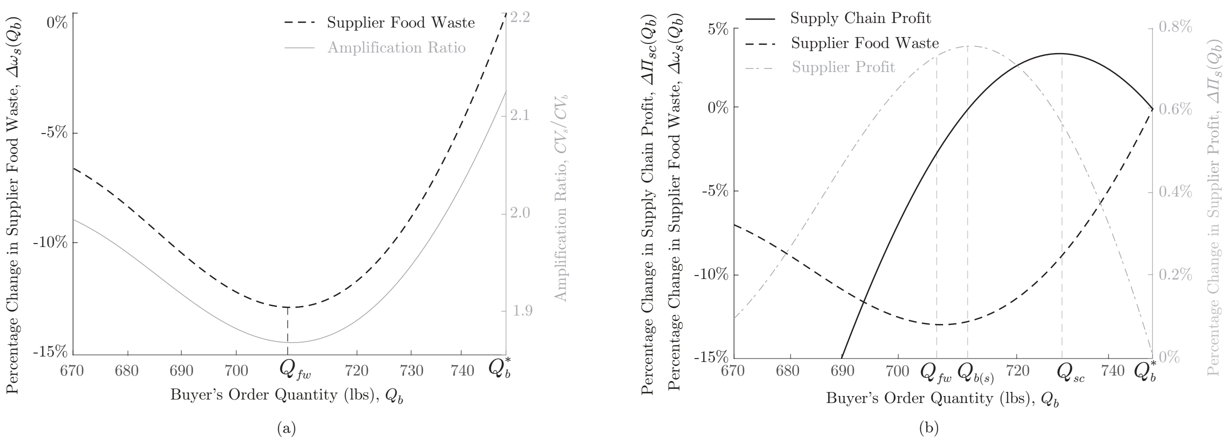
Note: Drawn for the baseline value of all parameters except for the parameter varied on x-axis.

FIGURE 8.3. Impact on the Amplification Ratio CV_s/CV_b.

supplier demand variability CV_s is increasing in the standard deviation of product expiration time σ_{T_b} (see the callout in Panel b), and as a result, the amplification ratio is also increasing.

Similarly, when it comes to the impact of the standard deviation of the buyer demand σ_d, in local markets and/or for expensive perishable products, we observe an initial decrease of CV_s followed by an increase. However, this is not the case for the baseline parameters where the replenishment travel distances are substantial; the supplier demand variability CV_s is increasing in the standard deviation of buyer demand σ_d (see the callout in Panel c). Interestingly, despite the increasing CV_s, we still see the decreasing/increasing pattern in the amplification ratio (see Panel c). Higher standard deviation of buyer demand also increases the buyer demand variability CV_b. First, this direct increase in CV_b with the increase in the standard deviation of buyer demand surpasses its indirect increase in supplier demand variability CV_s, leading to the decrease of the amplification ratio. Surprisingly, the indirect increase in CV_s eventually overtakes the direct increase in CV_b, which causes the eventual increase of the amplification ratio. Such an unexpected increase is due to the truncation effect (see section 6). Specifically, as the standard deviation of buyer demand becomes large enough, the truncation effect occurs for the long-inventory-depletion-time realizations, while at the same time, the extremely short-inventory-depletion-time realizations will be barely truncated. That is, not only the standard deviation of the buyer's replenishment cycle length increases but also its mean decreases. These two changes result in the eventual higher (indirect) increase in the supplier demand variability (see 5.1) than the (direct) increase in the buyer demand variability CV_b.

In sum, products with more extreme (high or low) purchasing price, replenishment cost, mean of product expiration time and standard deviation of buyer demand, as well as those with high mean buyer demand and standard deviation of product expiration time, are likely to exhibit higher extent of variability propagation. With this in mind, we can examine Figure 8.1. On the high amplification ratio side (high demand variability propagation): Cheddar's low standard deviation of buyer demand and high standard deviation of expiration time both contribute to the resulting high variability. Banana's low standard deviation of buyer demand and mean of expiration time, as well as high standard deviation of expiration time and mean of buyer demand all conspire to yield high variability propagation. On the low amplification ratio side (low demand variability propagation): Milk's high standard deviation of buyer demand and low standard deviation of expiration time both contribute to low variability. While, juice's high standard deviation of buyer demand also



Note: the percentage change in supplier food waste $\Delta w_s(Q_b) \equiv (\omega_s(Q_b) - \omega_s(Q_b^*)\omega_s(Q_b^*)^{-1}$,the percentage change in supplier profit $\Delta\Pi_s(Q_b) \equiv (\Pi_s(Q_b) - \Pi_s(Q_b^*)\Pi_s(Q_b^*)^{-1}$, the percentage change in supply chain profit (i.e., the supplier's and buyer's total profits) $\Delta\Pi_{sc}(Q_b) \equiv (\Pi_{sc}(Q_b) - \Pi_{sc}(Q_b^*))\Pi_{sc}(Q_b^*)^{-1}$. The figure is drawn for cucumber, $x = 150$ and $z = 1000$ (Table 2). The time limit in the supplier warehouse is, on average, 20% of the total shelf life, $T_s = \frac{0.2}{1-0.2}\mu_{T_b}$(Kinarm, 2015; Fusin, 2017; Caemin Industries, 2018), $p_s = 0.8p_b$ (Leonard, 2019).

FIGURE 8.4. Supplier Food Waste, Profit and Supply Chain Profit.

leads to low amplification ratio, its higher standard deviation of expiration time increases variability propagation, resulting in juice's higher variability amplification ratio than that of milk.

8.2. **Supply Chain Coordination.** We have shown that for perishable products, the extent of the upstream variability propagation is modulated by buyer's order quantity (see section 6). That is, by adjusting the buyer's order quantity, the supplier can experience less demand variability, which in turn could generate less food waste and higher profits for both the supply chain and the supplier itself. We investigate this next.

Figure 8.4 demonstrates how the percentage changes of the supplier food waste ω_s, supplier profit Π_s, and supply chain profit Π_{sc} vary with respect to the buyer's order quantity Q_b.

Figure 8.4 (a) shows that the change in supplier food waste (black dashed curve) closely traces the pattern of supplier demand variability or, equivalently, that of the variability amplification ratio (grey solid curve) since the buyer demand variability is unaffected by the order quantity. Intuitively, a decrease in supplier demand variability (i.e., the anti-bullwhip-like effect) results in better inventory management, which reduces supplier food waste. Furthermore, our numerical examination of a variety of perishable products (Table 2) reveals that such reduction of the supplier food waste could be substantial, potentially as much as 53.5%, with a typical increase in three to twenty-five percent range. For example, for cucumber (Figure 8.4 a) if the buyer places an order quantity that minimizes supplier food waste Q_{fw}, the supplier could reduce its food waste by about

13.1% as compared to that at the buyer's optimal order quantity. Similarly, for orange juice, the supplier could reduce its food waste by 20.4% if the buyer ordered supplier-food-waste-minimizing order quantity.

Besides, the possible decrease in supplier food waste, arising from the anti-bullwhip-like effect, can further translate into an increase in the supplier profit. Specifically, in Figure 8.4 (b), we observe that the change in supplier profit (grey dash-dot line), roughly follows (in the opposite direction) the change in the supplier food waste (black dashed curve). To explain this, the supplier profit is determined by the tradeoff between its food waste cost and revenue from selling to the buyer. In the beginning, both the decrease in supplier food waste and the increase in sales revenue (by selling more with higher Q_b) contribute to the initial fast increase in supplier profit. Subsequently, the supplier profit increases more slowly, because its food waste starts to increase, yet revenue from more sales still surpasses the increase in food waste. Finally, the supplier food waste increases drastically, which in turn exceeds the sales revenue gain, and as a result, the supplier profit decreases. The percentage change in supplier profit can also be considerable as compared to that at the buyer's optimal order quantity. Amongst the products considered (Table 2) we observed an increase of as much as 42.5%, with a typical increase in one to six percent range. For the cucumber, if the buyer purchases the quantity that maximizes supplier profit $Q_{b(s)}$ (Figure 8.4 b), the supplier could increase its profit by around 0.74% as compared to that at the buyer's optimal order quantity. For orange juice, the supplier could increase its profit by 5.24% if the buyer purchases supplier-profit-maximizing order quantity.

Nevertheless, such adjustments of buyer's order quantity away from buyer's optimal order quantity naturally come at a price of lowering the buyer profit. Interestingly, as the black solid curve in Figure 8.4 (b) shows, there exists a buyer's order quantity that allows for higher supply chain profit together with lower supplier food waste, lower buyer food waste and higher supplier profit. In particular, decreasing the buyer's optimal order size Q_b^* to the quantity that maximizes the total supply chain profit Q_{sc} (Figure 8.4 b) will boost the supply chain profit by more than 3.3%. Importantly, it also reduces supplier food waste by 8.9% and buyer food waste by 38.3%. Since this deviation of the buyer's order quantity from the optimal lowers the buyer profit, an appropriate contract would be required for the redistribution of the gains, so that the system can be better off both in terms of its environmental impact and profit.

9. Extension

9.1. **Multiple Buyers.** Our analysis so far examined a supplier selling to a single buyer. We now consider a supplier selling to multiple buyers. Each of the n (identical) buyers faces the customer demand μ and orders the quantity of Q_b from the supplier.

Proposition 4. *The supplier demand variability is*

$$CV_s(Q_b, n) = \sqrt{\frac{1+(n-1)\rho}{n}} \sqrt{\frac{var\left[CL_b(Q_b)\right]}{E\left[CL_b(Q_b)\right]}}. \tag{9.1}$$

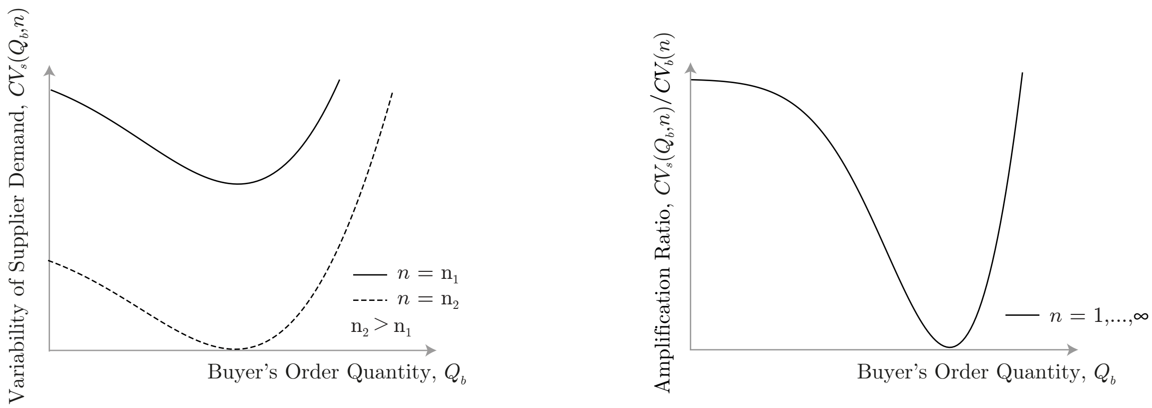
Here ρ is the correlation coefficient for buyer i's and buyer j's demand $\forall i \neq j$.

This proposition provides an expression for the supplier demand variability $CV_s(Q_b, n)$. Recall that $CV_s(Q_b) = \sqrt{\frac{var[CL_b(Q_b]}{E[CL_b(Q_b)]}}$ is the supplier demand variability in the one buyer supply chain as derived in section 4(see equation (5.1)). Thus, the supplier demand variability with multiple buyers can be written as $CV_s(Q_b, n) = \sqrt{\frac{1+(n-1)\rho}{n}} CV_s(Q_b)$. Note everything that we discussed in section 6, the truncation effect, etc., applies to each individual buyer and, thus, to $CV_s(Q_b)$. This means that the truncation effect and its resulting first-decreasing-then-increasing pattern for $CV_s(Q_b, n)$ also hold when there are multiple buyers.

We next look at the extent of the truncation effect and how it changes with the number of buyers and the correlation coefficient. Regardless of the number of buyers, each individual buyer's replenishment cycle length mean and variance (and thus $CV_s(Q_b)$) remain the same. Therefore, it is easy to see from equation 9.1 that, as expected, as the number of buyers increases, the supplier demand becomes less variable with $\rho < 1$ (see Figure 9.1 a) and unchanged with $\rho = 1$. This is simply due to the well-known statistical economies of scale. That is, the decrease in the supplier demand variability is simply due to the decrease in the total buyer demand variability with an increase in the number of buyers n: $CV_b^{tot}(n) = \sqrt{\frac{1+(n-1)\rho}{n}} CV_b$ (recall CV_b is the variability of the demand of one of the buyers). The amplification ratio allows us to account for this classic reduction in the buyer demand variability.

Proposition 5. *The variability amplification ratio $AR = \frac{CV_s(Q_b,n)}{CV_b^{tot}(n)} = \frac{CV_s(Q_b)}{CV_b}$ is independent of the number of buyers n.*

We find that the amplification ratio is independent of the number of buyers and is exactly equal to the amplification ratio in one buyer supply chain. In other words, the extent of variability reduction



(a) Supplier Demand Variability, $\rho < 1$

(b) Variability Amplification Ratio

FIGURE 9.1. The Impact of the Number of Buyers n

is independent of the number of buyers (see Figure 9.1 b). Thus, as compared with the single buyer case, multiple buyers do not change the extent of the variability propagation, and *all our findings directly apply.*

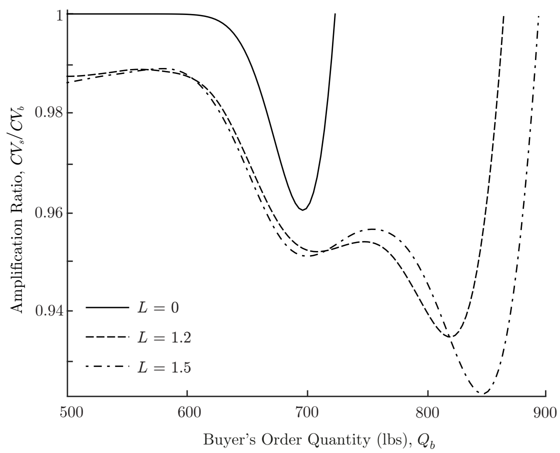
9.2. **Lead Time.** For analytical tractability, we assumed no lead time between placing and receiving an order for the buyer. We now consider a fixed and positive procurement lead time, denoted as L. The products are sold from stock to satisfy the buyer demand according to the FIFO policy. All unmet demand is lost. Following the literature, we employ the following continuous-review lot size-reorder point inventory control policy (Berk and Gürler, 2008).

Policy: *The buyer places a replenishment quantity Q_b whenever the inventory position hits r by demand, or drops to zero by perishing, whichever occurs first.*

Figure 9.2 (a) illustrates how buyer's order quantity changes the variability amplification ratio CV_s/CV_b with lead time L. Both the bullwhip-like and anti-bullwhip-like effects still arise, but the effect of buyer's order quantity turns out to be a bit more intricate with the positive lead time. Such a complicated effect arises from the possibility of stockout during lead time and its interaction with the truncation effect.

The following four scenarios capture all possibilities of when product expiration happens (t_x inventory depletion time of x units):

(1) No product expiration happens (all Q_b units of product are sold before expiration): $t_{Q_b} \leq T_b$ (no truncation, possible stockout).



The figure is drawn for all parameters set to those of cucumber, $x = 150$ and $z = 1000$ (Table 2).

FIGURE 9.2. Variability Amplification Ratio with Lead Time.

(2) Product expiration of current batch happens after new batch arrives: $t_{Q_b - r} + L \leq T_b < t_{Q_b}$ (truncation, no stockout)

(3) Product expiration of current batch happens before new batch arrives but after placing new order: $t_{Q_b - r} \leq T_b < t_{Q_b - r} + L$ (truncation is substituted by the stockout).

(4) Product expiration of current batch happens before placing new order (before inventory drops down to r): $t_{Q_b - r} > T_b$ (truncation + lead time long stockout).

In the first scenario, though the product does not expire and there is no truncation due to expiration, stockout arising from selling all remaining r units during lead time might happen and will play a role in determining cycle time. In the second scenario, truncation induced by product expiration solely determines cycle time (no stockout happens). In scenario three, while expiration happens, it does not determine the cycle length. The cycle length is solely determined by the stockout/lead time. In scenario four, truncation again plays a role in determining cycle length, together with the lead time. As buyer's order quantity Q_b increases, its inventory depletion time t_{Q_b}naturally becomes longer. That results in the likelihood associated with each of the four scenarios shifting more and more from the earlier ones to the later ones.

The first scenario is dominant (i.e., most likely to occur) when Q_b is sufficiently low. Here we observe a slight increase in the amplification ratio (see Figure 9.2, Q_b less than approx. 600) due to the potential stockout. Recall from equation 5.1 that supplier demand variability, and thus, amplification ratio, is determined by the variance and the mean of cycle length. On the one hand, when no stockout arises in this scenario, the buyer's cycle length is exactly the inventory depletion time t_{Q_b}, resulting in a constant amplification ratio. On the other hand, when stockout happens, the buyer's cycle length becomes $t_{Q_b-r}+L$, i.e. the inventory depletion time of reaching the reorder point plus the lead time. Note that the depletion time entirely determines the variance of cycle length (there is no lead time variability), while partially shaping the mean (due to the lead time L component) — this holds back the increase of cycle length mean with an increase in Q_b, contributing to an increase of the amplification ratio.

Once Q_b is sufficiently high, scenario two comes into play and we observe the decrease in the amplification ratio due to the truncation effect (see Figure 9.2, Q_b from approx. 600 to 700). However, once Q_b is sufficiently high for scenario three to become dominant, we once again see the increase in the amplification ratio (see Figure 9.2, Q_b from approx. 700 to 750) due to the stockout (like in scenario one). Finally, when Q_b is sufficiently high for scenario four to be prevalent, the first-decreasing-then-increasing pattern in the amplification ratio resumes due to the truncation effect (see Figure 9.2, Q_b more than approx. 750).

In sum, as Q_b increases, the stockout initially plays a role by slowing down the rate of increase of cycle length mean, leading to the initial increase of amplification ratio (see Figure 9.2, Q_b less than approx. 600). With further increase in the order quantity Q_b the truncation effect comes into play, leading to a first-decreasing-then-increasing pattern of amplification ratio. However, the decreasing path of the truncation effect gets interrupted by the third scenario, where stockout due to expiration during lead time happens and again results in a brief increase in the amplification ratio.

Comparing across different lengths of the lead time in Figure 9.2 (dashed vs. dash-dotted line), we observe that the lengths of the lead time L affect the amplification ratio.[5] Mostly, a longer lead time results in a lower amplification ratio. This happens because (whenever lead time plays a role) longer lead time extends the cycle length mean more than the shorter one (while keeping variance unchanged) – this leads to a considerably lower amplification ratio (see equation 5.1) with longer lead times. However, due to possible stockout in scenario one (see Figure 9.2, Q_b less than approx. 600) and scenario three (see Figure 9.2, Q_b from approx. 700 to 750), as discussed above, the longer lead time L slows down the increase of cycle length mean with an increase in Q_b more than the shorter lead time, leading to a faster increase of the amplification ratio (dashed line increases faster than the solid line). This generates a slightly higher amplification ratio with longer lead times in

[5]To ensure that we compare apples to apples, we adjust the reorder point to reflect the change in the lead time: $r = \mu L + z\sigma_d\sqrt{L}$, where z is the z-score corresponding to 95% service level in Figure 9.2.

these two regions. In addition, the extent of the amplification ratio decrease (from the left highest to the lowest) is about 36.3% higher for $L = 1.2$ as compared to the zero lead time (see Figure 9.2, dashed line vs. solid line, respectively). In other words, positive lead time causes a more pronounced truncation effect in the supply chains.

In sum, with positive lead time truncation effect operates like we discussed in the main paper, with slight interruption of the first-decreasing-then-increasing pattern resulting from the potential stockouts during the lead time. Furthermore, we observe that overall longer lead times generate lower amplification ratio.

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